# Introduction to OFDM

Fire Tom Wada
Professor, Information Engineering, Univ. of the Ryukyus
Chief Scientist at Magna Design Net, Inc
wada@ie.u-ryukyu.ac.jp
http://www.ie.u-ryukyu.ac.jp/~wada/

# What is OFDM?

- OFDM
  =Orthogonal Frequency Division Multiplexing

- Many orthogonal sub-carriers are multiplexed in one symbol
  - What is the orthogonal?
  - How multiplexed?
  - What is the merit of OFDM?
  - What kinds of application?

# Outline

- Background, history, application
- Review of digital modulation
- FDMA vs. Multi-carrier modulation
- Theory of OFDM
- Multi-path
- Summary

# Why OFDM is getting popular？

- State-of-the-art high bandwidth digital communication start using OFDM
  - Terrestrial Video Broadcasting in Japan and Europe
  - ADSL High Speed Modem
  - WLAN such as IEEE 802.11a/g/n
  - WiMAX as IEEE 802.16d/e

- Economical OFDM implementation become possible because of advancement in the LSI technology

# Japan Terrestrial Video Broadcasting service

- ISDB-T (Integrated Services Digital Broadcasting for Terrestrial Television Broadcasting)
- Service starts on 2003/December at three major cities (Tokyo, Nagoya, Osaka)
- Full service area coverage on 2006
- 5.6MHz BW is divided into 13 segments (~430KHz BW)
- HDTV: 12 segments
- Mobile TV : 1 segment
- SDTV: 4 segment
- Analog Service will end 2011

# Brief history of OFDM

- First proposal in 1950’s
- Theory completed in 1960’s
- DFT implementation proposed in 1970’s
- Europe adopted OFDM for digital radio broadcasting in 1987
- OFDM for Terrestrial Video broadcasting in Europe and Japan
- ADSL, WLAN(802.11a)

# Digital modulation basics

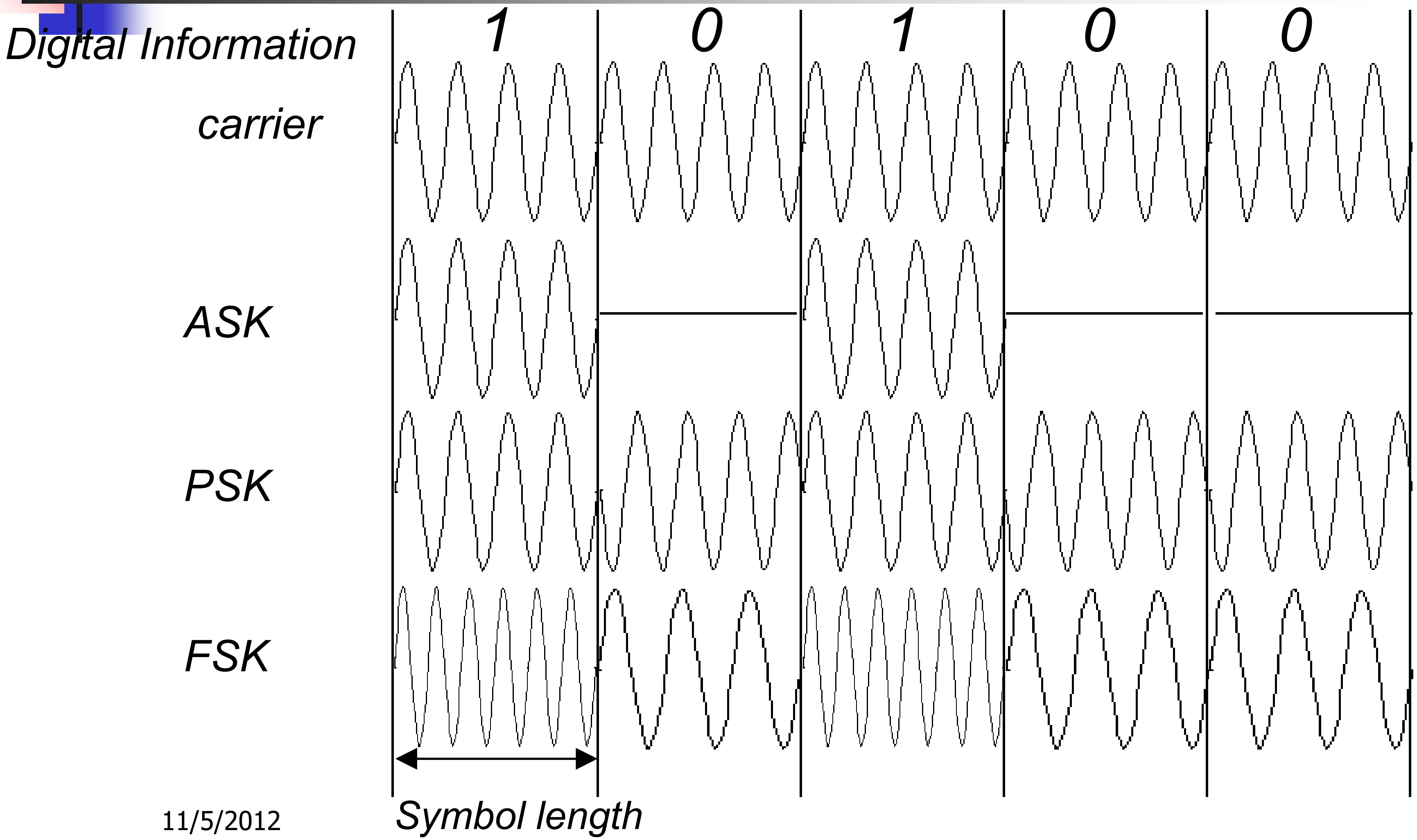
- Digital modulation modulates three parameters of sinusoidal signal.
- A, $\theta_k$ fc,

$$s(t) = A \cdot \cos(2\pi \cdot f_c \cdot t + \theta_k)$$

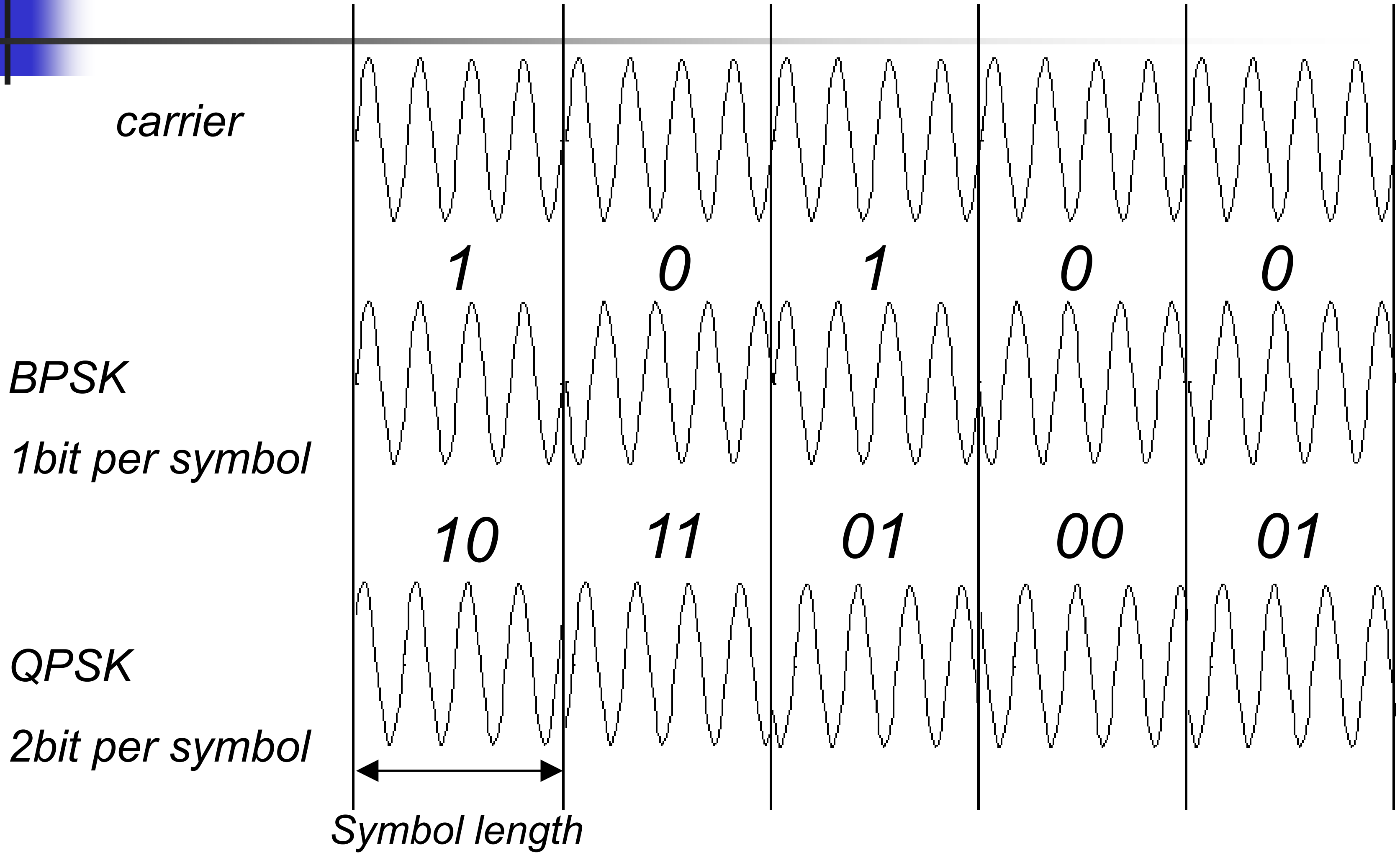
- Three type digital modulation:
  - ASK : Amplitude Shift Keying
  - PSK : Phase Shift Keying
  - FSK : Frequency Shift Keying

***OFDM uses combination of ASK and PSK such as QAM, PSK***

# Symbol Waveform



# Multi bit modulation



# Mathematical expression of digital modulation

- Transmission signal can be expressed as follows

$$s(t) = \cos(2\pi \cdot f_c \cdot t + \theta_k)$$

$$= \cos\theta_k \cdot \cos(2\pi \cdot f_c \cdot t) - \sin\theta_k \cdot \sin(2\pi \cdot f_c \cdot t)$$

$$a_k = \cos\theta_k, \quad b_k = \sin\theta_k$$

$$s(t) = \mathrm{Re}[(a_k + jb_k)e^{j2\pi fc \cdot t}]$$

- s(t) can be expressed by complex base-band signal $(a_k + jb_k)e^{j2\pi fc \cdot t}$

$e^{j2\pi fc \cdot t}$ *Indicates carrier sinusoidal*

$(a_k + jb_k)$ *Digital modulation*

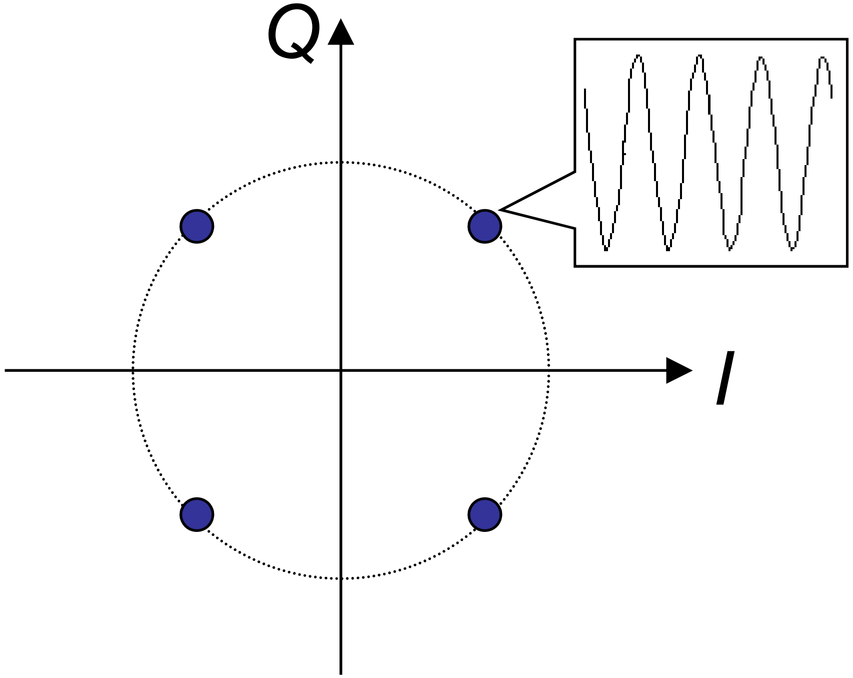
***Digital modulation can be expressed by the complex number***

# Constellation map

- $(a_k + jb_k)$ is plotted on I(real)-Q(imaginary) plane

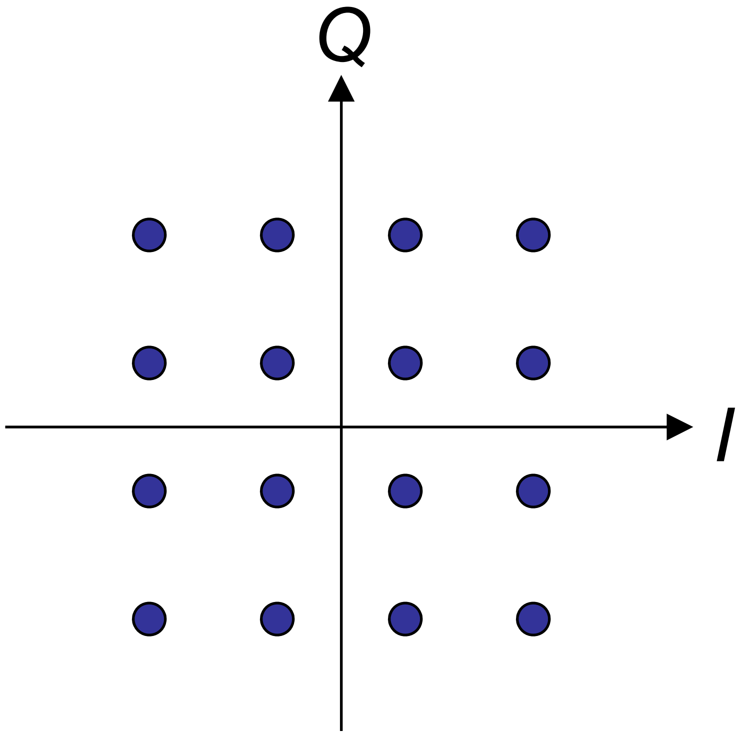
| data | | $a_k$ | $b_k$ |
|---|---|---|---|
| 00 | π/4 | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| 01 | 3π /4 | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| 11 | 5π /4 | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ |
| 10 | 7π /4 | $\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ |

*QPSK*

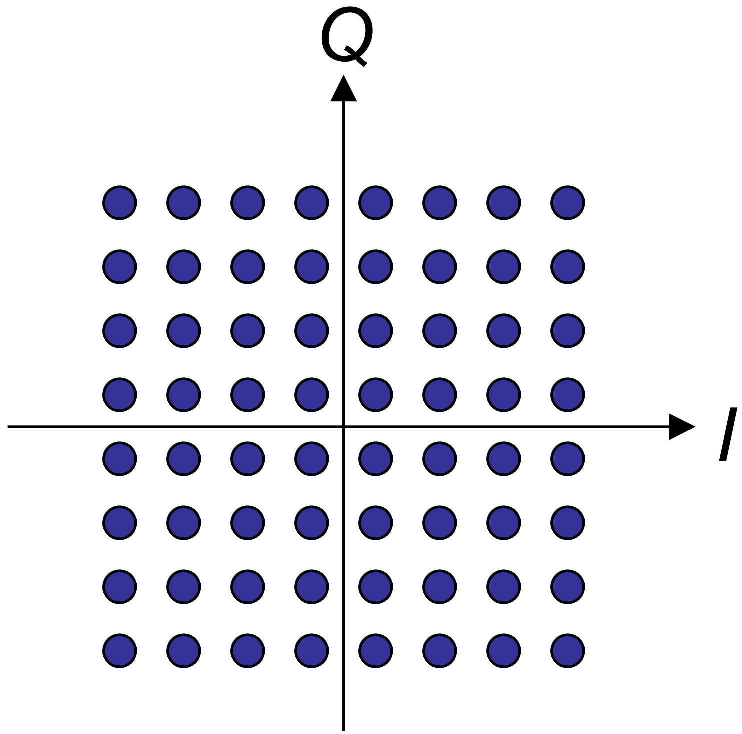


# Quadrature Amplitude Modulation (QAM)

*16QAM*




*64QAM*

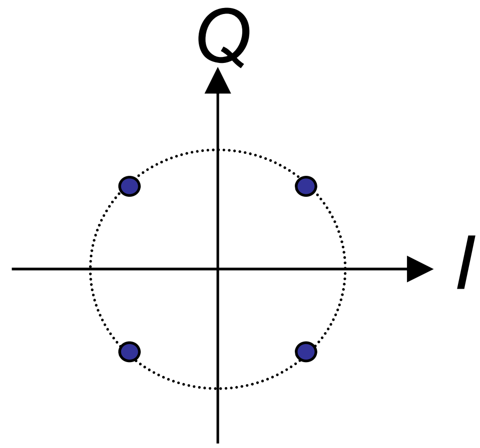


# Summary of digital modulation

- Type of modulation: ASK,PSK,FSK,QAM
- OFDM uses ASK,PSK,QAM
- Digital modulation is mathematically characterized by the coefficient of complex base-band signal

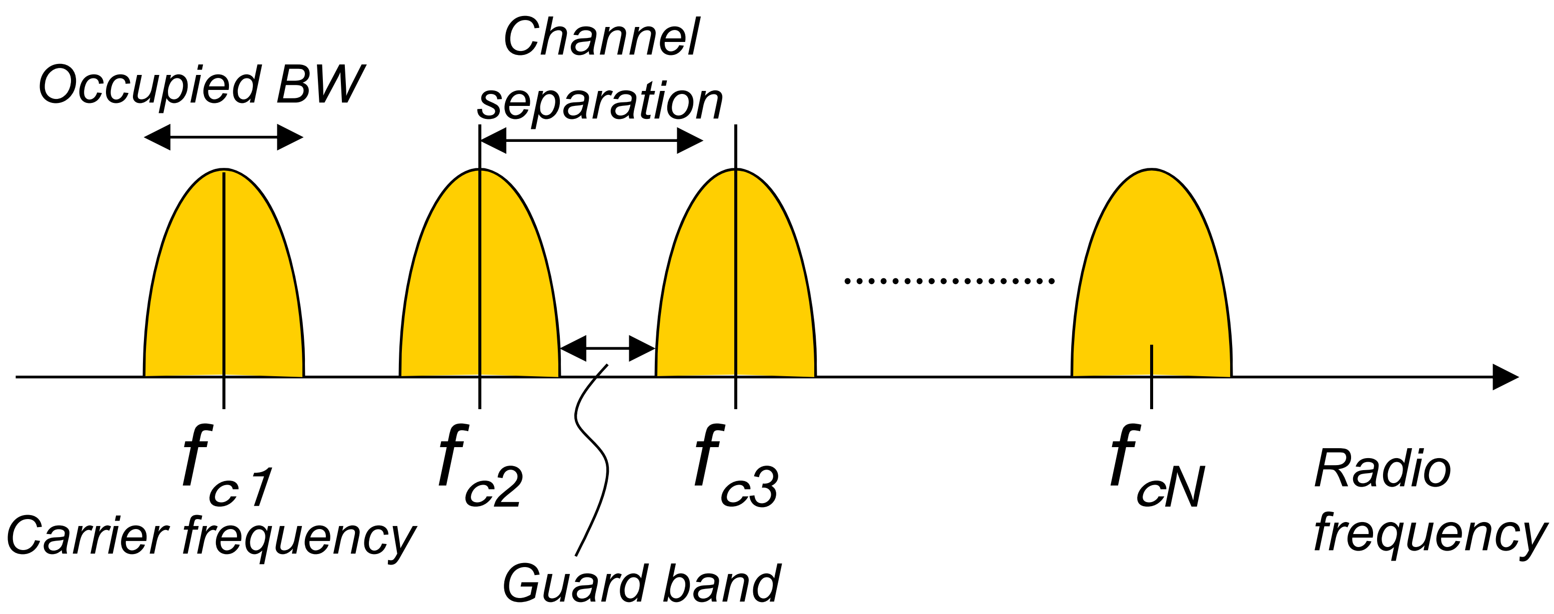
$$(a_k + jb_k)$$

- Plot of the coefficients gives the constellation map



# Frequency Division Multiple Access (FDMA)

- Old conventional method (Analog TV, Radio etc.)
- Use separate carrier frequency for individual transmission



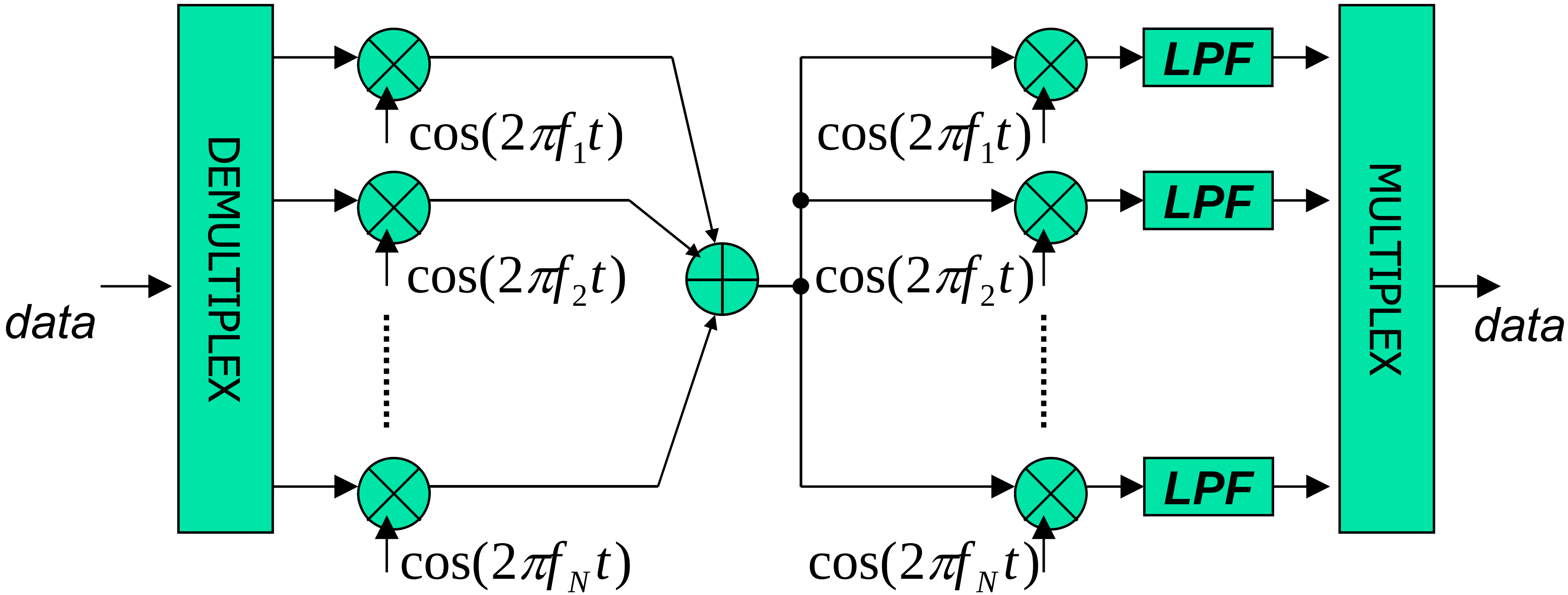
# Japan VHF channel assignment

| Channel number | Frequency (MHz) |
|---|---|
| 1 | 90-96 |
| 2 | 96-102 |
| 3 | 102-108 |
| 4 | 170-176 |
| 5 | 176-182 |
| 6 | 182-188 |
| 7 | 188-194 |
| 8 | 192-198 |
| 9 | 198-204 |
| 10 | 204-210 |
| 11 | 210-216 |
| 12 | 216-222 |

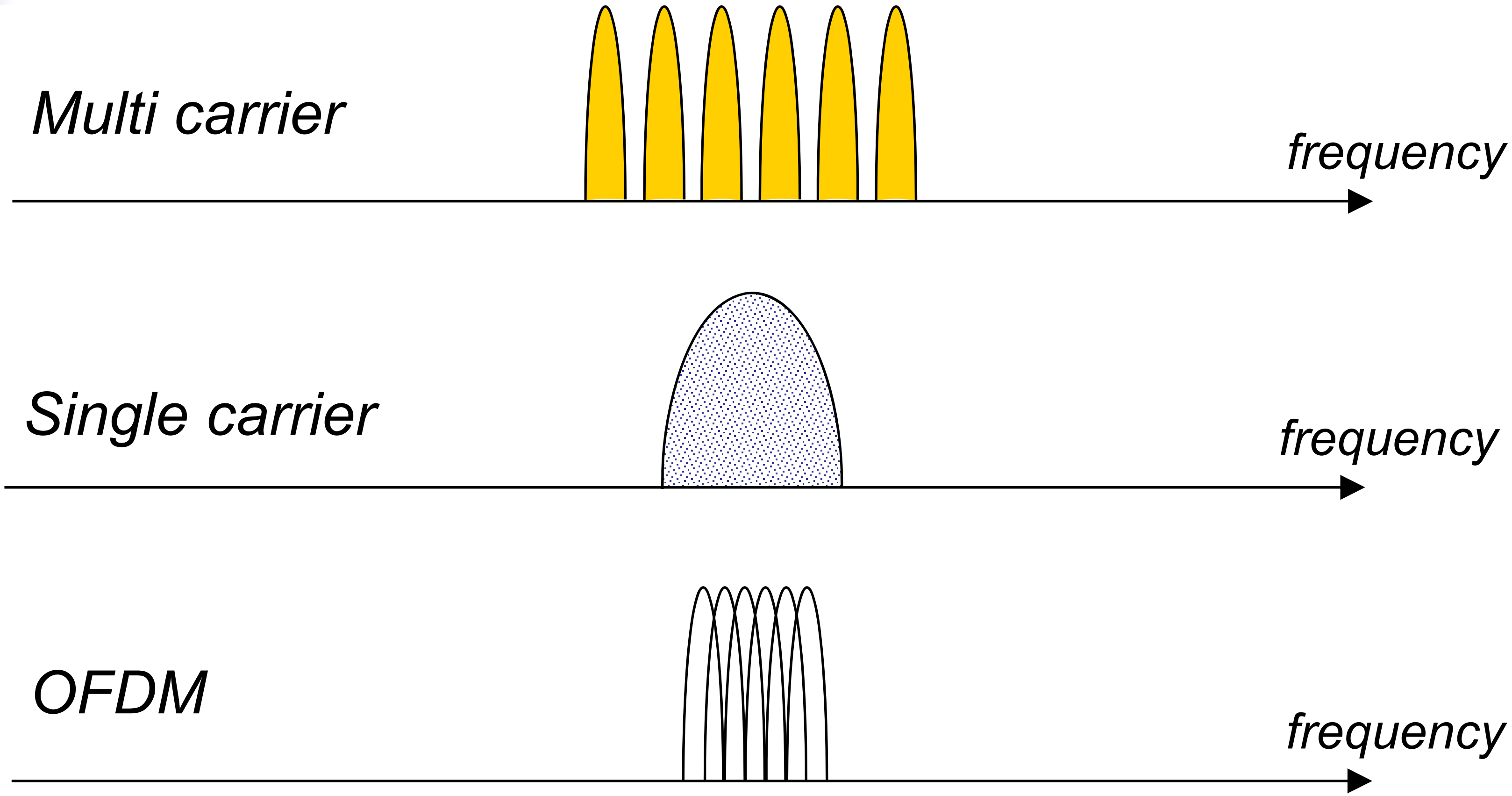
- Channel Separation = 6MHz

# Multi-carrier modulation

- Use multiple channel (carrier frequency) for one data transmission



# Spectrum comparison for same data rate transmission



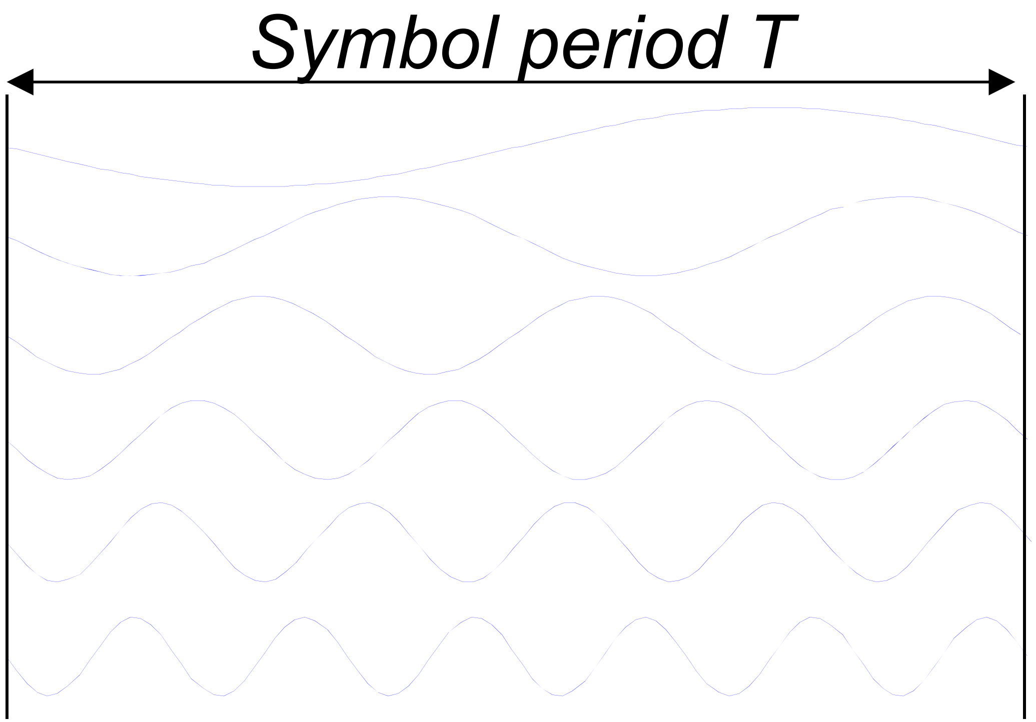
# OFDM vs. Multi carrier

- OFDM is multi carrier modulation
- OFDM sub-carrier spectrum is overlapping
- In FDMA, band-pass filter separates each transmission
- In OFDM, each sub-carrier is separated by DFT because carriers are orthogonal
  - Condition of the orthogonality will be explained later
- Each sub-carrier is modulated by PSK, QAM

***Thousands of PSK/QAM symbol can be simultaneously transmitted in one OFDM symbol***

# OFDM carriers

- OFDM carrier frequency is n·1/T




$$f_0 = \frac{1}{T}$$

$$\cos(2\pi \cdot 1 \cdot f_0 \cdot t + \theta_1)$$

$$\cos(2\pi \cdot 2 \cdot f_0 \cdot t + \theta_2)$$

$$\cos(2\pi \cdot 3 \cdot f_0 \cdot t + \theta_3)$$

$$\cos(2\pi \cdot 4 \cdot f_0 \cdot t + \theta_4)$$

$$\cos(2\pi \cdot 5 \cdot f_0 \cdot t + \theta_5)$$

$$\cos(2\pi \cdot 6 \cdot f_0 \cdot t + \theta_6)$$

# Sinusoidal Orthogonality

- m,n: integer, $T=1/f_0$

$$\int_0^T \cos(2\pi m f_0 t)\cdot\cos(2\pi n f_0 t)dt = \begin{cases} \dfrac{T}{2} & (m=n) \\ 0 & (m\neq n) \end{cases}$$ $\longrightarrow$ Orthogonal

$$\int_0^T \sin(2\pi m f_0 t)\cdot\sin(2\pi n f_0 t)dt = \begin{cases} \dfrac{T}{2} & (m=n) \\ 0 & (m\neq n) \end{cases}$$ $\longrightarrow$ Orthogonal

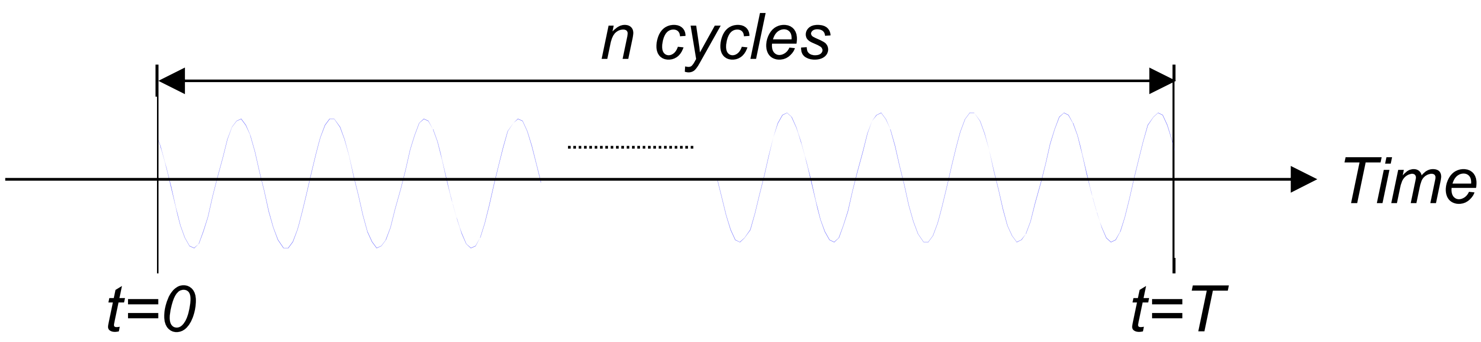
$$\int_0^T \cos(2\pi m f_0 t)\cdot\sin(2\pi n f_0 t)dt = 0$$ $\longrightarrow$ Orthogonal

# A sub-carrier of f=nf$_0$

$$a_n \cdot \cos(2\pi n f_0 t) - b_n \cdot \sin(2\pi n f_0 t)$$

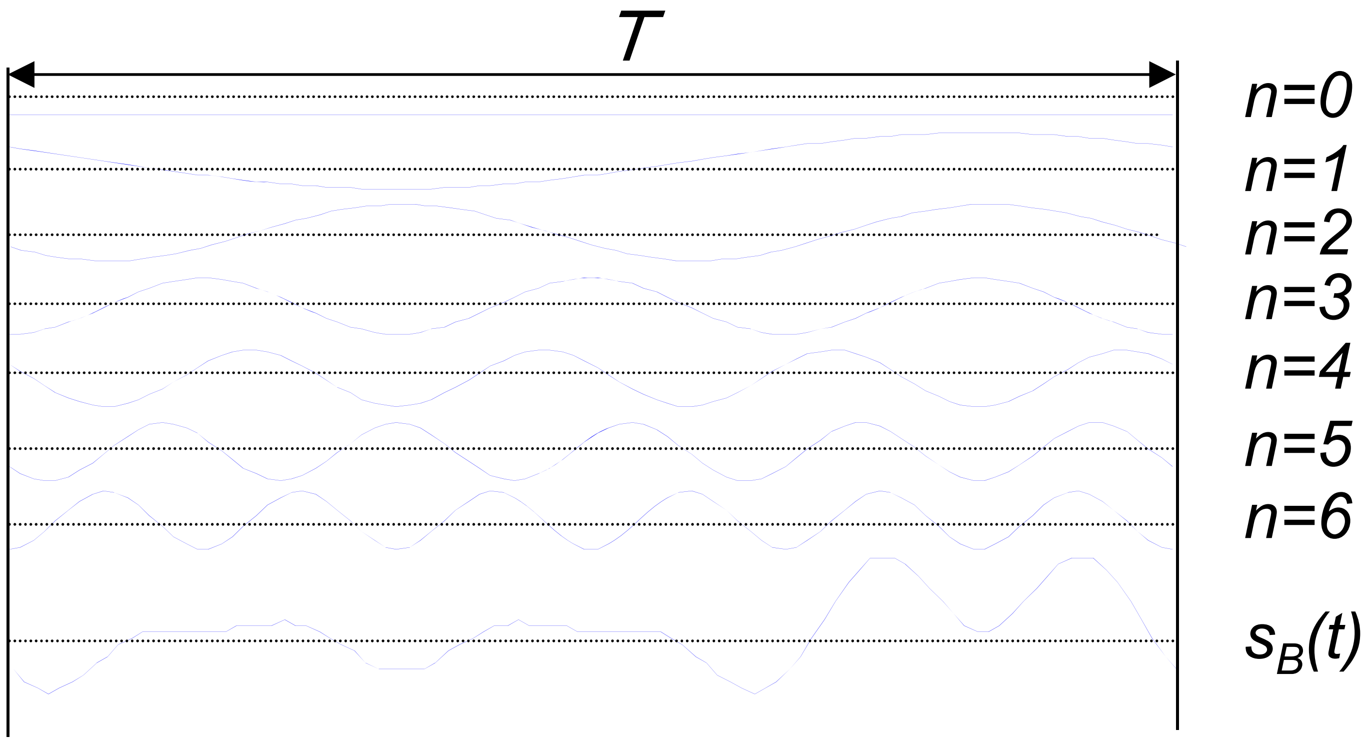
$$= \sqrt{a_n^{\,2} + b_n^{\,2}} \cos(2\pi n f_0 t + \phi_n), \qquad \phi_n = \tan^{-1} \frac{b_n}{a_n}$$

- Amplitude and Phase will be digitally modulated



# Base-band OFDM signal

$$s_B(t) = \sum_{n=0}^{N-1} \left\{ a_n \cos(2\pi n f_0 t) - b_n \sin(2\pi n f_0 t) \right\}$$



# How $a_n$,$b_n$ are caluculated from $s_B(t)$ - Demodulation Procedure -

$$\int_0^T s_B(t)\cdot\cos(2\pi k f_0 t)dt$$

$$=\sum_{n=0}^{N-1}\left\{a_n\int_0^T \cos(2\pi n f_0 t)\cos(2\pi k f_0 t)dt - b_n\int_0^T \sin(2\pi n f_0 t)\cos(2\pi k f_0 t)dt\right\}$$

$$=\frac{T}{2}a_k$$

$$\int_0^T s_B(t)\{-\sin(2\pi k f_0 t)\}dt=\frac{T}{2}b_k$$

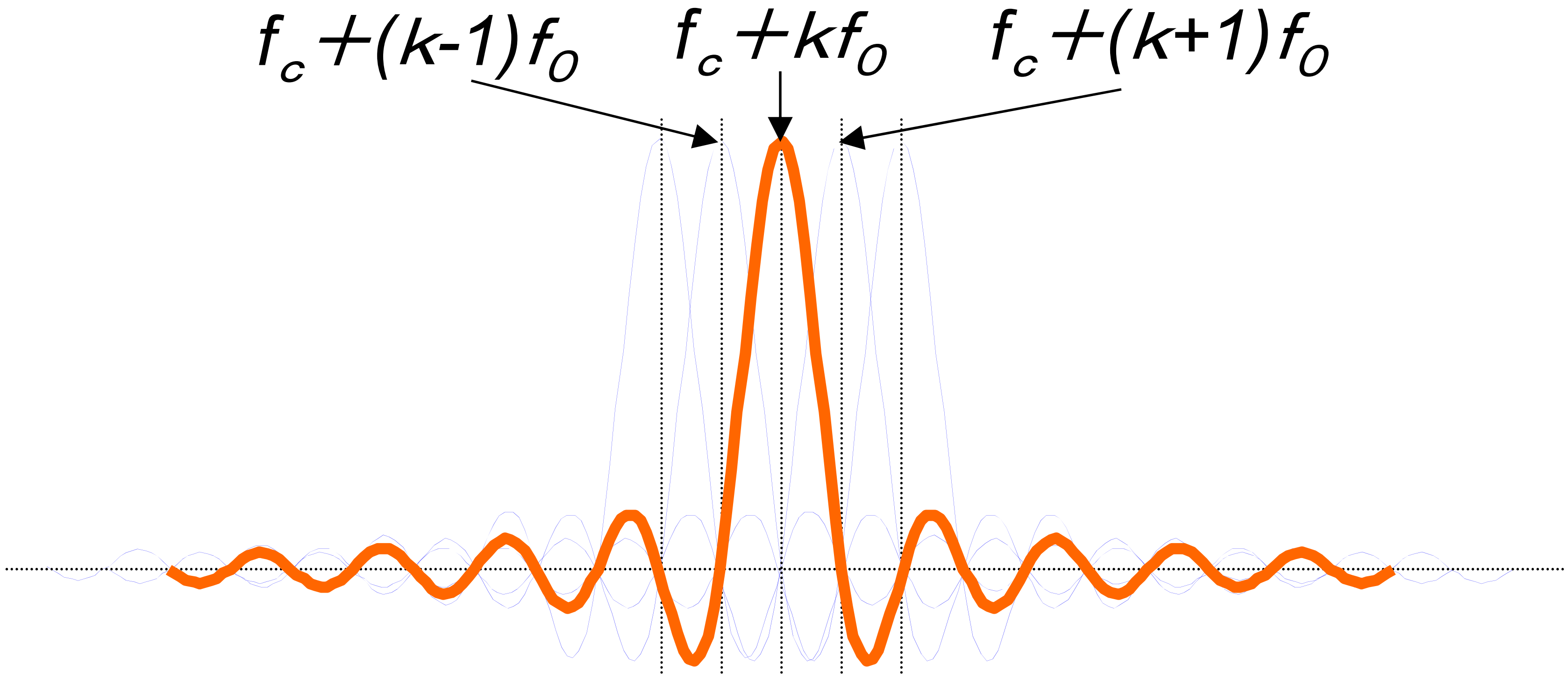
- According to the sinusoidal orthogonality, $a_n$,$b_n$ can be extracted.
- In actual implementation, DFT(FFT) is used
- N is roughly 64 for WLAN, thoudand for Terrestrial Video Broadcasting

# Pass-band OFDM signal

- *$S_B(t)$ is upcoverted to pass-band signal $S(t)$*
- *$f_c$ frequency shift*

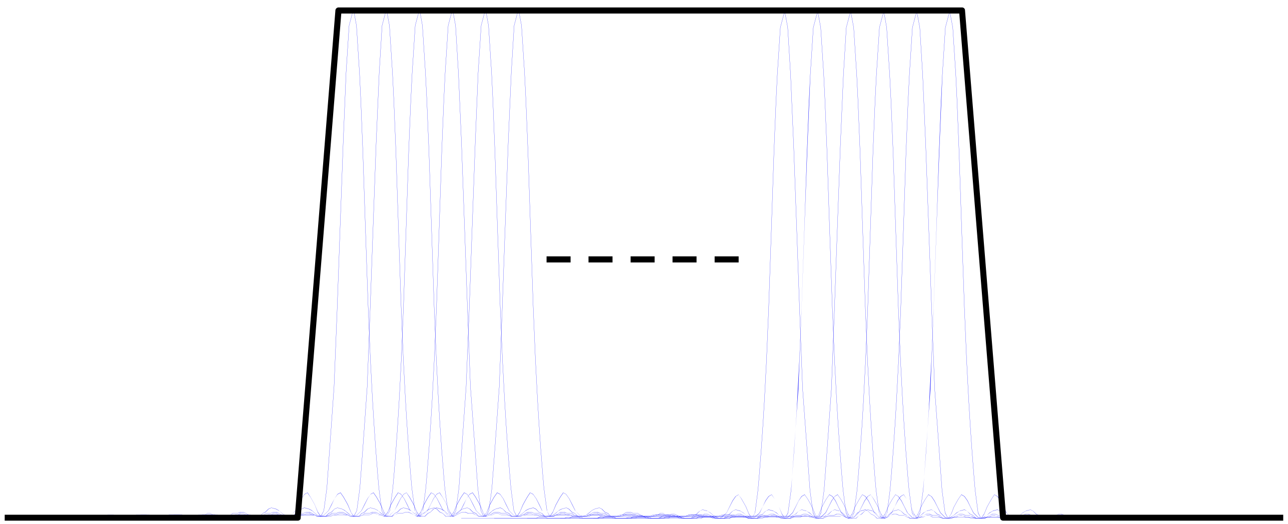
$$s(t)=\sum_{n=0}^{N-1}\left[a_n\cos\left\{2\pi(f_c+nf_0)t\right\}-b_n\sin\left\{2\pi(f_c+nf_0)t\right\}\right]$$

# Actual OFDM spectrum



# OFDM power spectrum

- Total Power spectrum is almost square shape



# OFDM signal generation

$$s(t)=\sum_{n=0}^{N-1}\left[a_n\cos\left\{2\pi(f_c+nf_0)t\right\}-b_n\sin\left\{2\pi(f_c+nf_0)t\right\}\right]$$

- Direct method needs

  N digital modulators

  N carrier frequency generator

  ➔ Not practical
- In 1971, method using DFT is proposed to OFDM siganal generation

# OFDM signal generation in digital domain

- Define complex base-band signal u(t) as follows

$$s_B(t) = \mathrm{Re}\left[u(t)\right]$$

$$u(t) = \sum_{n=0}^{N-1} d_n \cdot e^{j2\pi n f_0 t}, \qquad d_n = a_n + jb_n$$

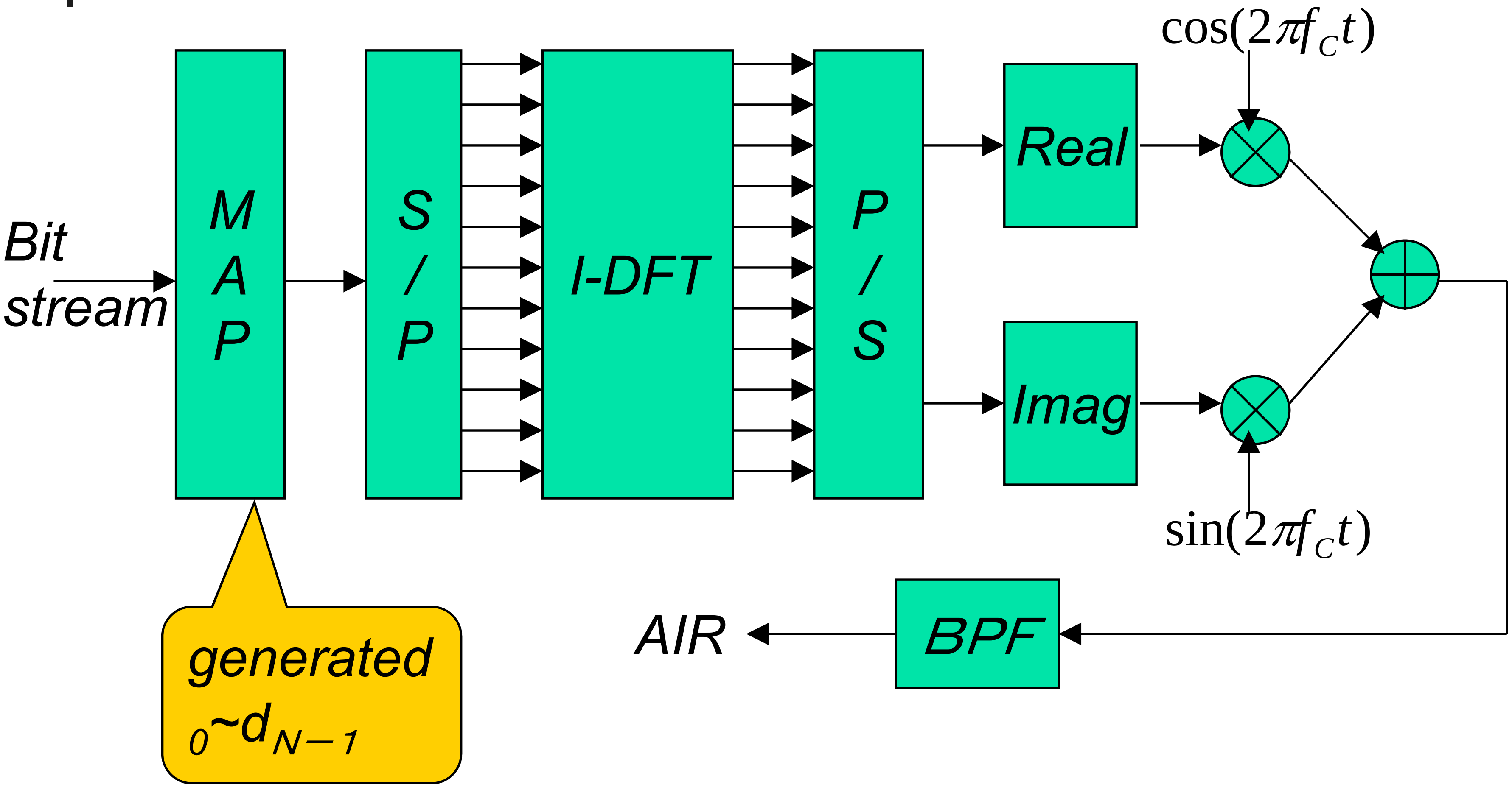
- Perform N times sampling in period T

$$u\left(\frac{k}{Nf_0}\right) = \sum_{n=0}^{N-1} d_n \cdot e^{j2\pi n f_0 \frac{k}{Nf_0}} = \sum_{n=0}^{N-1} d_n \cdot e^{j\frac{2\pi nk}{N}}$$

$$= \sum_{n=0}^{N-1} d_n \cdot \left(e^{j\frac{2\pi}{N}}\right)^{nk} \qquad (k = 0,1,2,\cdots, N-1)$$

$$u(k) = IFFT\ (d_n) = IFFT(a_n + jb_n)$$

# OFDM modulator



# OFDM demodulation

$$s(t)=\sum_{n=0}^{N-1}\left[a_n\cos\{2\pi(f_c+nf_0)t\}-b_n\sin\{2\pi(f_c+nf_0)t\}\right]$$

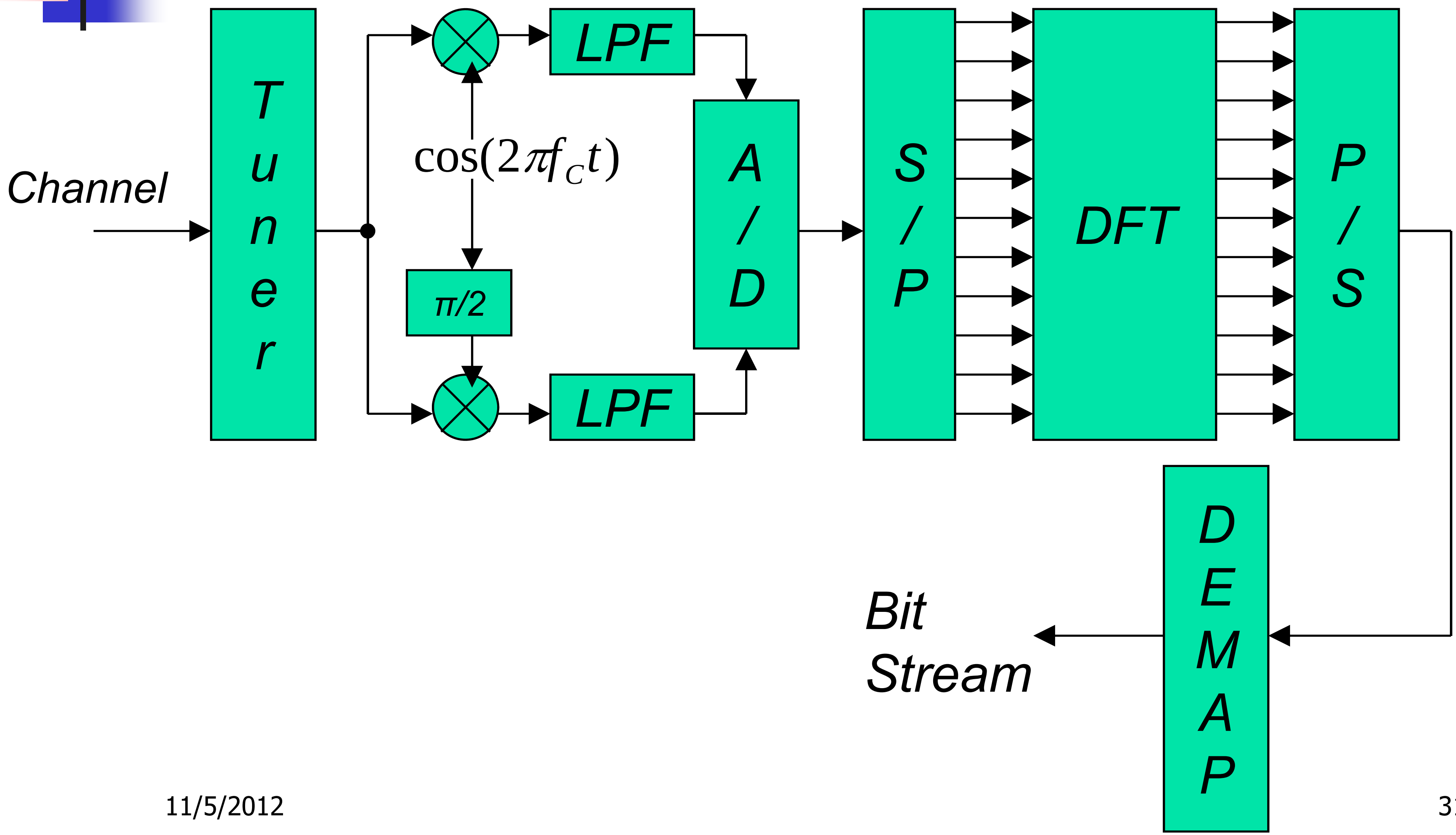
$$LPF\ [s(t)\cdot\cos(2\pi f_C t)]=\frac{1}{2}\sum_{n=0}^{N-1}\{a_n\cos(2\pi nf_0 t)-b_n\sin(2\pi nf_0 t)\}=\frac{1}{2}s_I(t)$$

$$LPF\ [s(t)\cdot\{-\sin(2\pi f_C t)\}]=\frac{1}{2}\sum_{n=0}^{N-1}\{a_n\sin(2\pi nf_0 t)+b_n\cos(2\pi nf_0 t)\}=\frac{1}{2}s_Q(t)$$

$$u(t)=s_I(t)+js_Q(t)=\sum_{n=0}^{N-1}d_n\cdot e^{j2\pi nf_0 t}$$

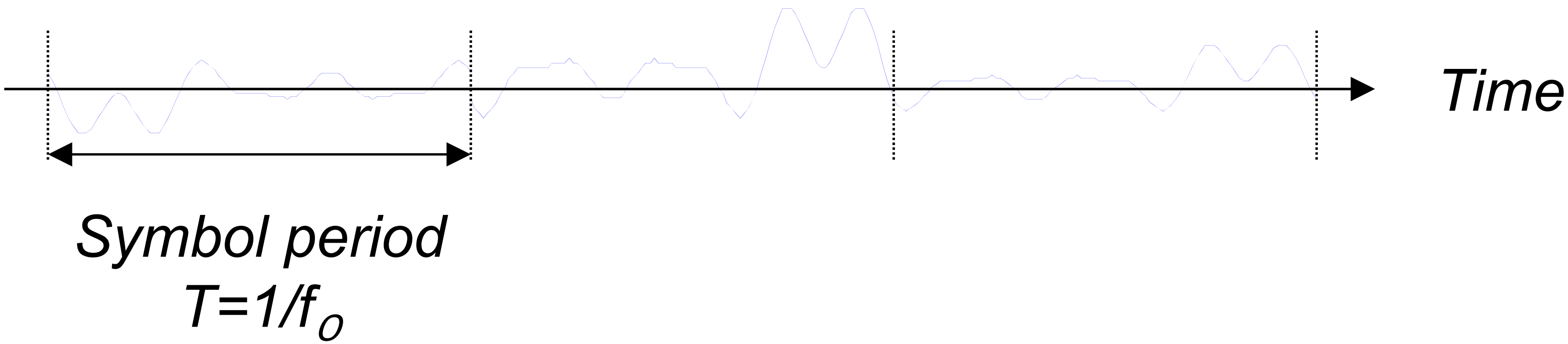
$d_n$ = *FFT(u(k))*

# OFDM demodulator (Too simple)



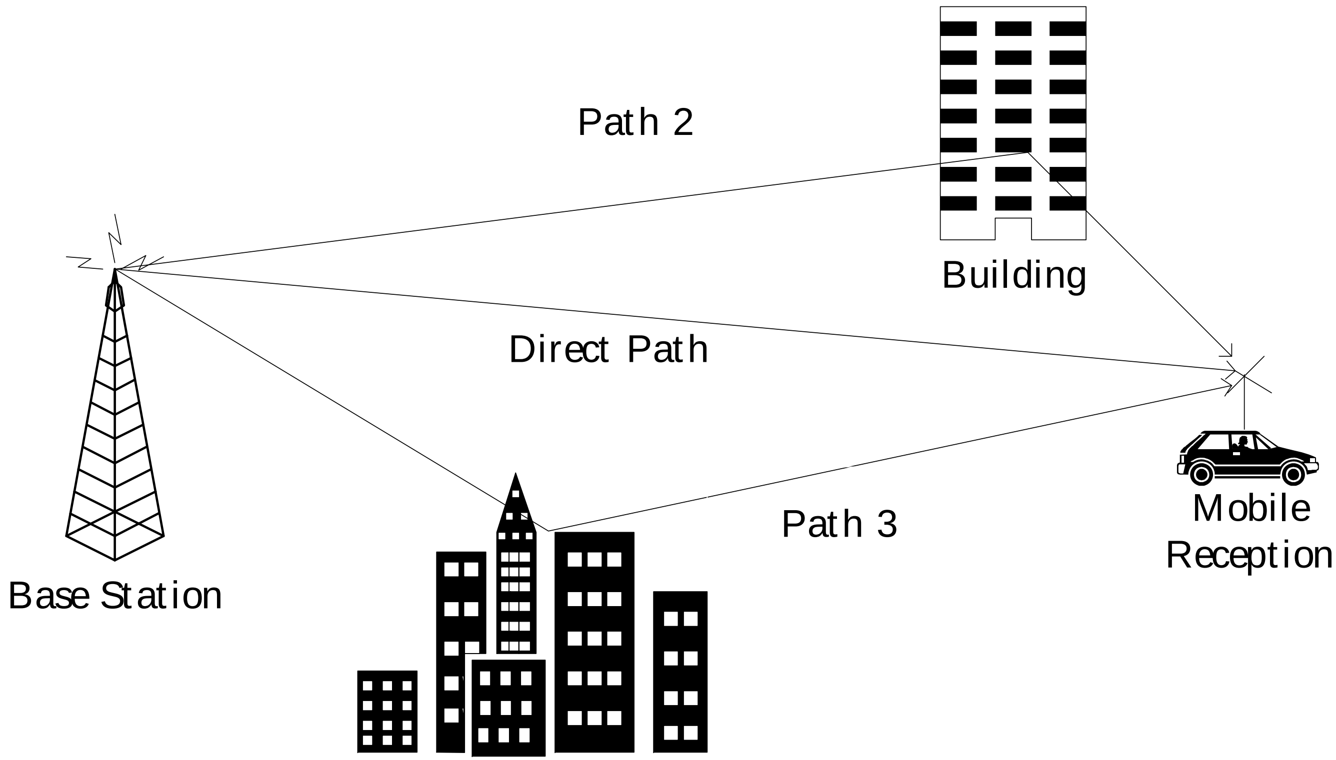
# Summary of OFDM signal

- Each symbol carries information
- Each symbol wave is sum of many sinusoidal
- Each sinusoidal wave can be PSK, QAM modulated
- Using IDFT and DFT, OFDM implementation became practical

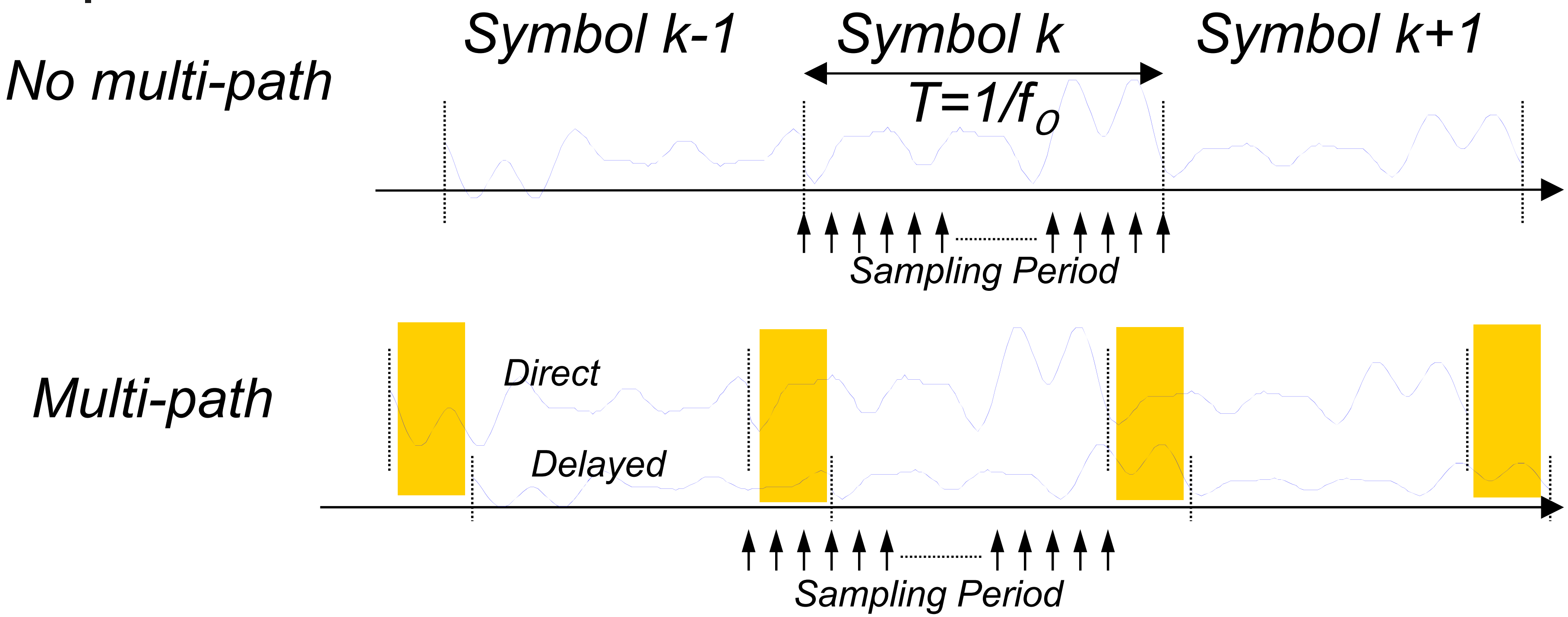


# Multi-path

- Delayed wave causes interference

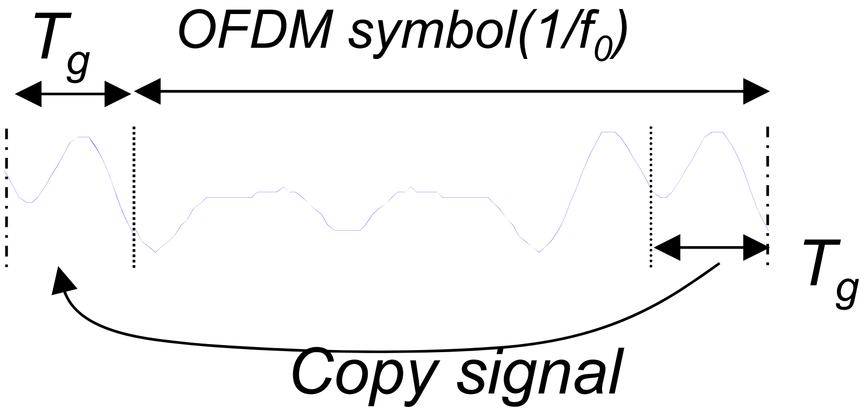


# Multi-pass effect

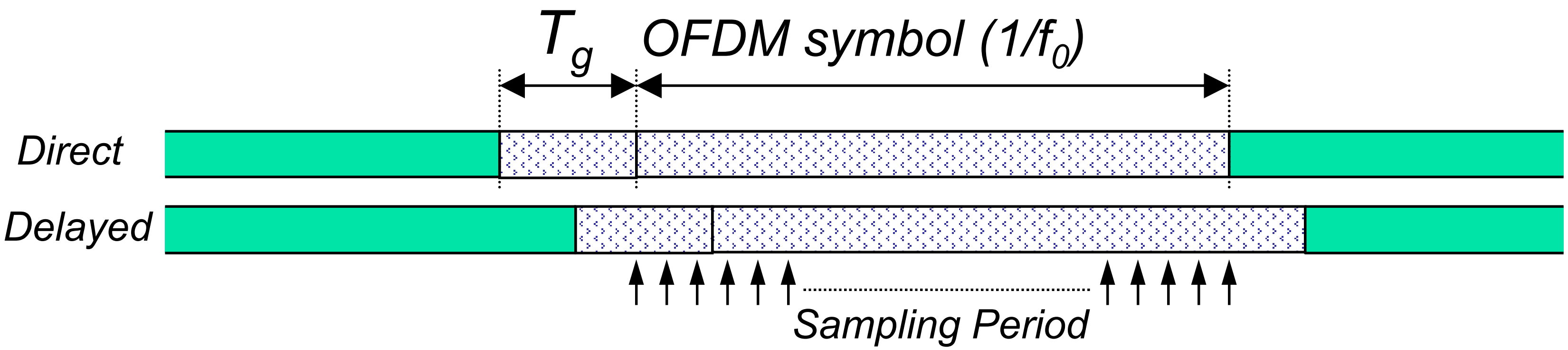



- Inter symbol interference (ISI) happens in Multi-path condition

# Guard Interval $T_g$

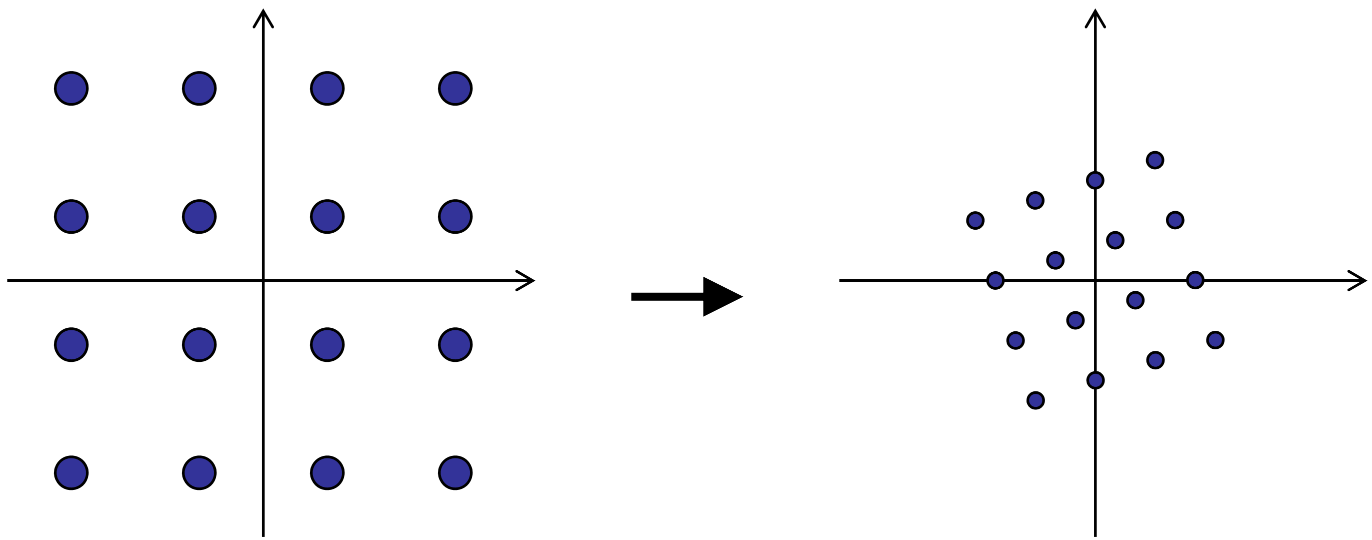



- By adding the Gurard Interval Period, ISI can be avoided

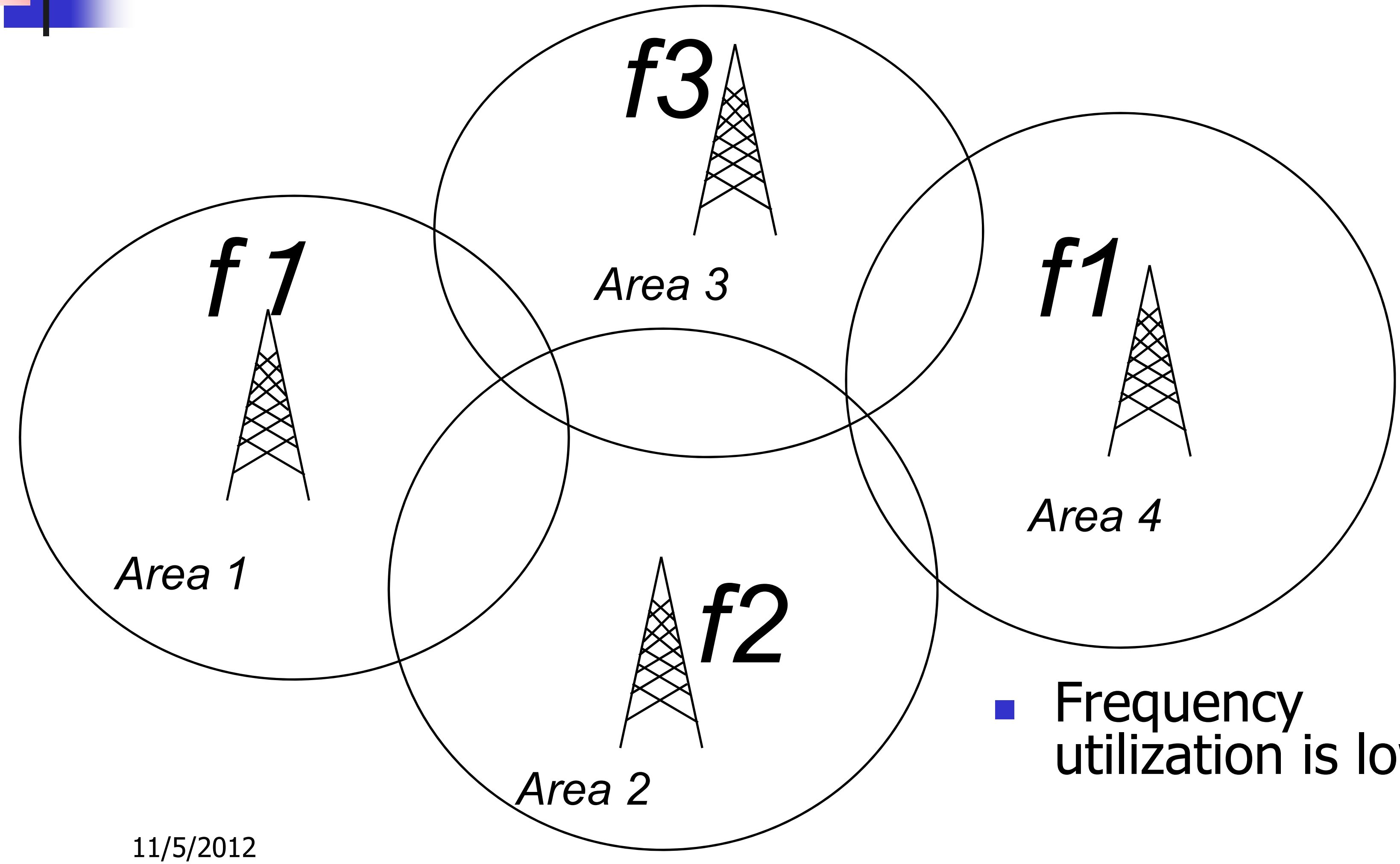


# Multi-path

- By adding GI, orthogonality can be maintained
- However, multi-path causes Amplitude and Phase distortion for each sub-carrier
- The distortion has to be compensated by Equalizer

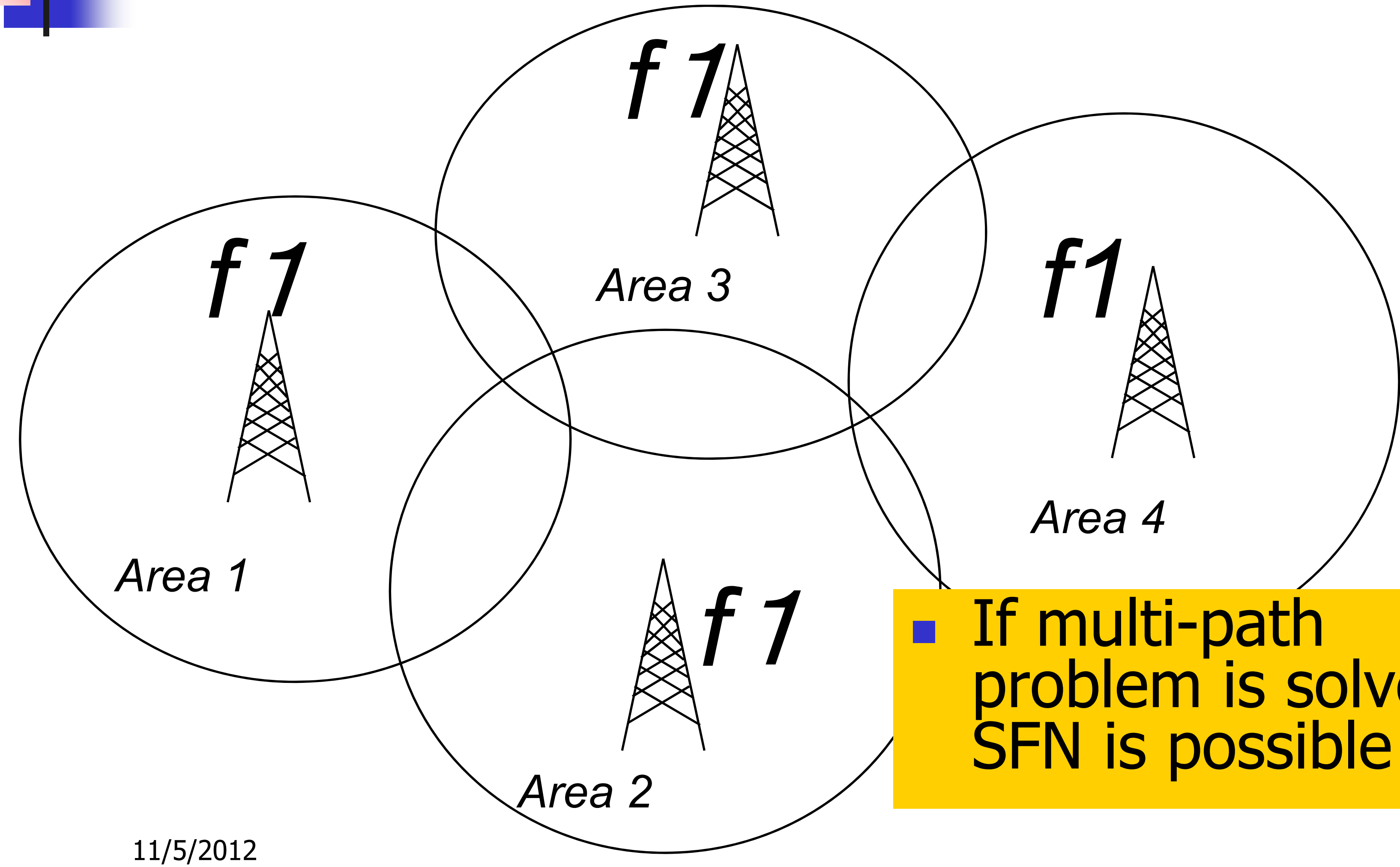


# Multiple Frequency Network




- Frequency utilization is low

# Single Frequency Network




- If multi-path problem is solved, SFN is possible

# That's all for introduction

- Feature of OFDM
  1. High Frequency utilization by the square spectrum shape
  2. Multi-path problem is solved by GI
  3. Multiple services in one OFDM by sharing sub-carriers (3 services in ISDB-T)
  4. SFN
  5. Implementation was complicated but NOW possible because of LSI technology progress